Abstract—Prices in electricity markets are given by the dual variables associated with the supply-demand constraint in the dispatch problem. However, in unit-commitment-based day-ahead markets, these variables are not easy to obtain. A common approach relies on re-solving the dispatch problem with the commitment decisions fixed, and utilizing the associated dual variables. This avenue may lead to inadequate revenues to generators, which has led to the introduction of uplift payments made by the market operator for further compensating the generators. An alternative pricing mechanism known as convex hull pricing has been proposed to reduce or eliminate uplift payments. Computation of these prices requires the global maximization of an associated Lagrangian dual problem. In this paper, we present an extreme-point-based procedure for obtaining a global maximizer. Unlike standard subgradient schemes where an arbitrary subgradient is used, we present an extreme-point subdifferential (EPSD) algorithm; this is a novel technique in which the steepest ascent direction is constructed by solving a continuous quadratic program. The EPSD algorithm initiates a move along this direction, employing an a priori constant steplength, with the intent of reaching the boundary of the face. A backtracking scheme selects a steplength that ensures descent with respect to a suitably defined merit function. As most electricity markets today co-optimize energy and reserves, an extension of the proposed convex hull pricing algorithm is provided for such integrated markets. Under suitable assumptions, we compare outcomes of energy-only and energy-reserve co-optimized markets under different pricing and uplift rules. In these examples, pricing rules have a major impact on the total payment while the uplift payment only accounts for a small portion of it. We also observe that it remains unclear whether marginal-cost pricing or convex-hull pricing leads to higher total payment.

Index Terms—Convex hull price, electricity markets, uplift payments, nondifferentiable optimization, Lagrangian relaxation, energy-reserve co-optimization, unit commitment.

I. INTRODUCTION

Currently, all day-ahead electricity markets in North America employ a unit-commitment-based model that explicitly incorporates each generator’s physical/operational constraints. Furthermore, the framework allows for start-up, no-load and marginal costs in the specification of offers. The operation and dispatch mechanisms of these markets are similar to those of a tightly regulated power pool: the quantity sold by each generator is determined by solving a centralized unit commitment and economic dispatch problem, except that the costs in this formulation are overridden by offer prices.

While the best way to determine prices remains an open question, a common practice is to derive prices, referred to as “marginal-cost prices”, from the solution of the corresponding economic dispatch problem in which commitment decisions are fixed. Note that these commitment decisions are derived from an a priori solution of a unit commitment problem. Start-up and no-load costs that are commitment-dependent are ignored when computing these prices. As a consequence, the payments collected through the auction may be insufficient to compensate the generators. To overcome this problem, uplift payment mechanisms have been introduced, through which additional side payments are made to the generators in recognition of the costs incurred due to commitment decisions.

The introduction of uplift payments brings potentially new difficulties for both the operator and the market participants. First, they may lead to allocations of payments that are neither transparent nor easy to justify. The strong dependence of marginal-cost prices on the commitment solution raises concerns regarding the equity, efficiency and economic rationale of electricity markets [1], [2]. As the size of the unit commitment problems often precludes computing exact solutions, the dual variables associated with the corresponding dispatch problems, parameterized by these commitment decisions, may vary dramatically with solution accuracy. Yet another difficulty is that the marginal-cost prices no longer increase monotonically with demand. In particular, a high price does not necessarily indicate a high level of demand, since committed generators with high offers may set the marginal-cost prices. Accordingly, prices fail to assume their usual economic roles as indicators of the alignment between supply and demand, or incentives for adjusting supply and consumption levels [3].

Motivated by the need to develop pricing and uplift models in the presence of discrete commitment decisions, several alternate payment mechanisms have been proposed. The discrete-decision-pricing approach, as proposed by O’Neill et al. [4], requires constraints be imposed to restrict the commitment decisions to the levels specified by solving the unit commitment

Current research in unit-commitment-based electricity markets focuses on the development of pricing models that capture important aspects of the market dynamics, such as the costs associated with commitment decisions, while also providing incentives for participants to engage in behaviors that are consistent with the overall objectives of the market. This paper presents an extreme-point subdifferential method for obtaining convex hull prices in electricity markets, which provide a framework for determining prices that are consistent with the goals of the market. The proposed method is based on an extreme-point subdifferential (EPSD) algorithm and is designed to be applied to day-ahead electricity markets where energy and reserve co-optimization is considered.

The convex-hull pricing approach is based on the principle that the cost of electricity is determined by the cost of the least expensive energy and reserve units that are committed to meet the demand. In order to determine the convex-hull price, the algorithm must find the extreme points of the convex hull of the feasible region, which is defined by the constraints of the market. The EPSD algorithm uses a backtracking scheme to select the steplength that ensures descent with respect to a suitably defined merit function. The algorithm then initiates a move along this direction, employing an a priori constant steplength, with the intent of reaching the boundary of the face. A backtracking scheme selects a steplength that ensures descent with respect to a suitably defined merit function. As most electricity markets today co-optimize energy and reserves, an extension of the proposed convex hull pricing algorithm is provided for such integrated markets. Under suitable assumptions, we compare outcomes of energy-only and energy-reserve co-optimized markets under different pricing and uplift rules. In these examples, pricing rules have a major impact on the total payment while the uplift payment only accounts for a small portion of it. We also observe that it remains unclear whether marginal-cost pricing or convex-hull pricing leads to higher total payment.

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Motivated by the need to develop pricing and uplift models in the presence of discrete commitment decisions, several alternate payment mechanisms have been proposed. The discrete-decision-pricing approach, as proposed by O’Neill et al. [4], requires constraints be imposed to restrict the commitment decisions to the levels specified by solving the unit commitment
problem, and uses the associated Lagrange multipliers from this restricted problem as the prices for discrete decisions. However, this mechanism leads to zero or near-zero auction surpluses for all players [5]. A continuous formulation was employed by New York ISO where commitment decisions are assumed to be continuous; however, the associated multipliers of the supply-demand constraint may still not support the equilibrium solution. An innovative market clearing scheme that guarantees non-negative auction surplus of generators has been recently proposed [6]. Under such pricing mechanisms, generators may still not maximize their auction surplus, leading to an opportunity cost. Nonlinear and non-anonymous pricing mechanisms, or generalized uplift mechanisms, have been proposed [7], [8]. Despite the mathematical elegance, the discriminatory and complex nature of these pricing mechanisms may lead to challenges in adoption within this industry.

Convex hull pricing has been introduced to address these undesirable properties of current pricing mechanisms [5], [9]. Convex hull prices do take into account start-up and no-load costs, without fixing commitment decisions a priori. The prices are non-decreasing with respect to demand, and lead to the minimal total “opportunity-cost” uplift payments [9]. In spite of a more complicated model, such pricing and uplift mechanisms are being seriously considered by the industry and may be implemented in the Midwest region of the US in the near future. In addition to energy, the system operator requires reserves to maintain system reliability. A wide consensus has emerged in academia and industry, suggesting the joint operation of energy-reserves markets, allowing for energy and reserves to be co-optimized.

Motivated by the poor local convergence behavior of existing computational schemes, the purpose of this paper is to provide a tool for computing convex-hull prices which were first proposed in [5], [9]. We develop a steepest ascent method (for piecewise linear concave maximization) that uses the extreme points associated with the solution set of a generator-specific auction-surplus-maximization problem to construct the subdifferential. As is customary in the literature (cf. [10]), these extreme points allow for formulating a quadratic program, whose solution represents the steepest ascent direction. Additionally, a backtracking scheme allows for determining a steplength that ensures descent with respect to a suitably defined merit function. We refer to our method as an Extreme-point subdifferential method (EPSD) for two reasons: (i) First, the subdifferential is constructed implicitly by aggregating the extreme points of the each individual generator’s best response to given prices. Working on extreme points of the smaller problems helps us to extract the subdifferential of $L(\rho)$. (ii) Second, our search mechanism is designed specifically to search over the extreme points of the faces of $L(\rho)$ which is a polyhedral function. While the scheme allows for moving to the interior of a face of $L(\rho)$, we explicitly leverage the polyhedrality of $L(\rho)$ in moving along the edges between faces through the backtracking line search. Numerical results show that the proposed method works well under the assumption that the single generator’s problem remains computationally tractable.

It should be emphasized that the proposed method bears a strong resemblance to bundle methods. Each extreme point of the subdifferential set (obtained from the aggregated auction-surplus-maximization quantity set) corresponds to a cutting plane at the point in question. In the rest of this paper, we use extreme points of the subdifferential set and cuts at the point in question interchangeably. Consequently, a bundle of cuts is constructed at each iterate. Furthermore, the construction of the subdifferential allows for coping with the nonsmoothness arising from the piecewise-linear nature of the dual problem. As one proceeds along the steepest ascent direction, cutting planes corresponding to new extreme points may be added to the bundle. In contrast, with most standard implementations, the steplength along this ascent direction is chosen so as to enable descent with respect to a prescribed merit function.

Extensions of this scheme to co-optimized markets are also provided. Note that under some assumptions, this scheme is shown to display a finite termination property and compares well with traditional subgradient schemes. Both aspects are discussed in a sequel to this paper (Part II).

The remainder of this paper contains six additional sections and is organized as follows. In Section II, we present a mathematical model for convex hull pricing in energy markets. Section III is devoted to analyzing the structure of the convex hull pricing problem. The structural characteristics of the problem are exploited in Section IV to develop an effective and efficient algorithm to compute convex hull prices for which a finite-termination convergence result is provided. Section V extends the algorithm to energy-reserve co-optimized markets. Economic analysis of market outcomes under convex-hull pricing scheme are presented in Section VI, and the paper concludes with some remarks and final thoughts in Section VII. Convergence analysis and numerical tests are provided in detail in Part II of this paper.

Before proceeding, we introduce the notation used throughout the paper.

$u_s$ commitment status vector of generator $s$

$p_s$ energy dispatch level vector of generator $s$

$f_s(u_s, p_s)$ offer function of generator $s$

$d_v(d)$ energy demand vector

$\nu(d)$ optimal unit commitment problem value function w.r.t. $d$

$v^h(d)$ convex hull of value function $\nu(d)$

$\rho^h(d)$ convex hull price vector

$\rho$ price vector

$\text{conv}(K)$ convex hull of set $K$

$L(\rho)$ Lagrangian dual function

$B_s(\rho)$ generator $s$’ auction-surplus-maximization quantity set under price $\rho$

$B(\rho)$ aggregated auction-surplus-maximization quantity set under price $\rho$

$\text{proj}_d(\rho)$ projection of $d$ onto $\text{conv}(B(\rho))$

$\gamma(\rho)$ merit function, or Euclidean distance between $d$ and $\text{conv}(B(\rho))$

$\nu$ index of iteration

$\Delta^\nu$ updating direction of iterate $\nu$

$\alpha^\nu$ steplength of iterate $\nu$
II. CONVEX HULL PRICING MODEL FOR ENERGY

In this section, we formulate a mathematical model for pricing and allocation in day-ahead energy markets, such as those currently prevalent in North America. On the basis of this model, we describe the determination of convex hull prices. The same model is subsequently extended to energy-reserve co-optimized markets in Section V.

Consider an \( H \)-period Day-Ahead Market (DAM) with \( S \) generators. Let \( f_s(u_s, p_s) \) be generator \( s \)'s offer function, where \( u_s \in \{0, 1\}^H \) denotes the commitment status of generator \( s \) (on or off) over the \( H \) periods. Furthermore, \( p_s \in ([0] \cup [p_s^{\text{min}}, p_s^{\text{max}}])^H \) denotes generator \( s \)'s energy dispatch levels, which could be zero (if off) or between \( p_s^{\text{min}} \) and \( p_s^{\text{max}} \) (if on) (the maximal and minimal power output levels of generator \( s \), both non-negative). Currently, all DAMs in the US adopt an offer format where \( f_s(u_s, p_s) \) is piecewise linear with respect to \( p_s \); we use the same offer conventions in our model.

Let \( X_s \) be the operational region defined by resource-based physical and/or operational constraints imposed on generator \( s \), which is typically characterized by polyhedral constraints and binary and continuous variables. Let \( d \in \mathbb{R}^H \) denote the demand vector over the \( H \) periods. Then, the Unit Commitment Problem (UCP) requires a set of commitment and dispatch decisions to satisfy the demand in the least “bid cost” manner, while being feasible with respect to physical and operational constraints.

**Definition 1 (UCP).** The UCP is defined as

\[
\min_{u, p} \quad \sum_{s=1}^{S} f_s(u_s, p_s) \\
\text{s.t.} \quad \sum_{s=1}^{S} p_s = d, \quad (u_s, p_s) \in X_s, \quad \forall s.
\]

We denote the value function by \( v(d) \): This is the optimal value of the mixed-integer linear program (1), parameterized by the demand \( d \).

A salient characteristic of the value function is that, on the set of \( d \) for which (UCP) is feasible, the value function is lower semicontinuous and differentiable almost everywhere. Indeed, the widely adopted marginal cost pricing model uses gradient or subgradient information associated with the value function as a candidate price.

On the other hand, the convex hull pricing scheme derives prices from the convex hull of the value function rather than the value function itself. The convex hull of a nonconvex function is the largest convex function that does not exceed the given function at any point in the domain [11]:

**Definition 2 (Convex Hull of the Value Function).** The convex hull of \( v(d) \) is defined as

\[
v^h(d) \triangleq \inf \{ \mu | (d, \mu) \in \text{conv}(\text{epi}(v(d))) \}, \tag{2}\]

where \( \text{epi}(f) \) is the epigraph of a function \( f \), and \( \text{conv}(K) \) denotes the convex hull of set \( K \).

A convex hull price is defined next.

**Definition 3 (Convex Hull Price).** A convex hull price, denoted by \( \hat{\rho}^h(d) \), is defined as a subgradient of the convex hull of the value function:

\[
\hat{\rho}^h(d) \in \partial v^h(d). \tag{3}\]

To simplify notation, we suppress the dependence of the convex hull price on \( d \) through the remainder of this paper.

![Fig. 1. value function and its convex hull](image)

We illustrate the value function and its convex hull for a 2-generator, 1-hour market in Figure 1. Note \( v(0) = 0 \) is part of the value function. In this example, the value function is defined on a discontinuous domain because of the minimal generating output constraints. The value function is characterized by nonconvexity and jump, due to the changes in commitment decisions as demand changes. The convex hull is the closest convex approximation from below.

III. ANALYSIS OF CONVEX HULL PRICING PROBLEM

In this section, we investigate the structural characteristics of the convex hull price problem for DAMs.

A. Convex Hull Price and Lagrangian Dual Problem

Obtaining the subdifferential of \( v^h(d) \) is a challenging proposition, since it necessitates computing the convex hull of a function. To exacerbatematters, every point in the hull requires the solution of a unit commitment problem, or effectively a mixed-integer linear program. We pursue an alternate tack based on solving the Lagrangian Dual Problem instead, which we define next.

**Definition 4 (Lagrangian Dual Problem of the UCP).** Suppose \( L(\rho) \) is defined as

\[
L(\rho) \triangleq \min_{(u_s, p_s) \in X_s, \forall s} \left\{ \sum_{s=1}^{S} f_s(u_s, p_s) + \rho^T \left( d - \sum_{s=1}^{S} p_s \right) \right\}.
\]

Then the Lagrangian dual problem is given by

\[
\max_{\rho} L(\rho). \tag{4}\]

The relationship between the Lagrangian dual problem and the original convex hull pricing problem is made precise by the next proposition [9], which is provided without a proof.

**Proposition 5.** Let \( \rho^* \) be an optimal solution to (4). Then

\[
\rho^* \in \partial v^h(d). \tag{5}\]
B. Characteristics of the Lagrangian Dual Problem

It is obvious that the Lagrangian dual function, \( L(\rho) \), is separable with respect to generators, and thus, the computation of \( L(\rho) \) can be reduced to a collection of \( S \) sub-problems with smaller scale. Furthermore, the Lagrangian dual function is concave and allows for the use of cutting plane methods. And the piecewise linear formal of the offer functions is a market design feature allows for the development of computational schemes. For the error analysis of such approximation, we refer the readers to [12].

In the Lagrangian dual problem, by relaxing the supply-demand balance constraint, the generators are in fact decoupled. Specifically,

\[
L(\rho) = \rho^T \mathbf{d} - \sum_{s=1}^{S} \max_{(\mathbf{u}_s, \mathbf{p}_s) \in \mathcal{X}_s} \left\{ \rho^T \mathbf{p}_s - f_s(\mathbf{u}_s, \mathbf{p}_s) \right\}.
\]

To facilitate our discussion, we define \( B_s(\mathbf{p}) \) as follows.

**Definition 6** (Generators’ Auction-Surplus-Maximization Quantity Set). Given a price vector \( \mathbf{p} \), the auction-surplus-maximization quantity set associated with generator \( s \) is defined as

\[
B_s(\mathbf{p}) \triangleq \left\{ \mathbf{p}_s | (\mathbf{u}_s, \mathbf{p}_s) \in \arg \max_{(\mathbf{u}_s, \mathbf{p}_s) \in \mathcal{X}_s} \left\{ \rho^T \mathbf{p}_s - f_s(\mathbf{u}_s, \mathbf{p}_s) \right\} \right\}.
\]

The optimization problem specified in (6) involves \( H \) binary and \( H \) continuous decision variables. Thus, we may use CPLEX [13], a commercial solver for large-scale mixed-integer linear and quadratic programs, capable of generating and storing multiple solutions, to obtain all extreme points of \( B_s(\mathbf{p}) \).

Note that while obtaining the extreme points is generally a challenging problem, we require these points only for a generator-specific problem, namely \( B_s(\mathbf{p}) \), and this problem is assumed to be tractable. We employ CPLEX as a proof of concept because it is typically easier to implement. In practice, dynamic programming (DP) may be a more computationally efficient choice for individual generator’s problem. Further, DP is capable of obtaining all discrete solutions because it enumerates all possible discrete states in each hour. Note \( B_s(\mathbf{p}) \) comprises generator \( s \)' dispatch levels, which is bounded.

Next, we define the aggregate quantity set \( B(\mathbf{p}) \).

**Definition 7** (Aggregated Auction-Surplus-Maximization Quantity Set). Given a price vector \( \mathbf{p} \), the aggregated auction-surplus-maximization quantity set is defined as

\[
B(\mathbf{p}) \triangleq \left\{ \sum_{s=1}^{S} \mathbf{p}_s | \mathbf{p}_s \in B_s(\mathbf{p}), s = 1, \ldots, S \right\}.
\]

The convex hull of \( B(\mathbf{p}) \) essentially captures the subdifferential of the dual function, which is leveraged by the proposed method. Note that \( B_s(\mathbf{p}) \) and \( B(\mathbf{p}) \) may have infinite cardinality, as a consequence of the degeneracies in the auction-surplus maximization problem. For example, if price coincides with marginal cost for a certain segment of a generator’s piecewise linear offer, then the generator is indifferent towards operating at any point within the segment. As a result, the generator’s auction surplus may be maximized over an interval, rather than a point. Fortunately, given the piecewise linear format, no matter what prices may be, the convex hull of the auction-surplus-maximization quantity sets has a finite number of extreme points. As a consequence, the maximization problem in (6) becomes a finite-dimensional linear program, once the commitment decisions, denoted by \( \mathbf{u}_s \), are given. The feasible region of such an LP is determined by \( u_s \) and \( X_s \), which is compact. Let \( \Theta(\mathbf{u}_s) \) denote the set of all vertices of the feasible region. Then, a solution to the linear program is attained at either a vertex, which is in \( \Theta(\mathbf{u}_s) \), or a convex combination of vertices, which is in \( \text{conv}(\Theta(\mathbf{u}_s)) \). Define \( \Phi_s \) as the union of vertices over all possible commitment schedule \( \mathbf{u}_s \) for generator \( s \) and \( \Phi \) be the aggregation of these vertices.

\[
\Phi_s \triangleq \cup_{\Theta(\mathbf{u}_s)} \text{ and } \Phi \triangleq \left\{ \sum_{s=1}^{S} \mathbf{p}_s | \mathbf{p}_s \in \Phi_s, \forall s \right\}, \quad (8)
\]

respectively. Next, we relate the convex hulls of \( B_s(\mathbf{p}) \) and \( B \) to \( B_s(\mathbf{p}) \) and \( B \cap \Phi \).

**Lemma 8.** Given bounded generating capacity, suppose \( \Phi_s \) and \( \Phi \) are defined by (8). Then, \( \Phi_s \) and \( \Phi \) have finite cardinality. Furthermore, \( \text{conv}(B_s(\mathbf{p})) = \text{conv}(B_s(\mathbf{p}) \cap \Phi_s) \) and \( \text{conv}(B(\mathbf{p})) = \text{conv}(B(\mathbf{p}) \cap \Phi) \).

**Proof:** The finiteness of \( \Phi_s \) follows from the finite number of vertices in a finite-dimensional LP. Since the number of commitment decisions is finite, \( \Phi_s \) is also finite since the union is over a finite set.

Since \( B_s(\mathbf{p}) \cap \Phi \subseteq B_s(\mathbf{p}) \), it follows that \( \text{conv}(B_s(\mathbf{p})) \) and \( \text{conv}(B_s(\mathbf{p}) \cap \Phi) \). Hence, \( \text{conv}(B_s(\mathbf{p}) \cap \Phi) \subseteq \text{conv}(B_s(\mathbf{p}) \cap \Phi) \). Remember \( B_s(\mathbf{p}) \cap \Phi \) gives all extreme-point solutions. By the fundamental theorem of linear programming, \( \text{conv}(B_s(\mathbf{p}) \cap \Phi) \subseteq \text{conv}(B_s(\mathbf{p}) \cap \Phi) \). Consequently, \( \text{conv}(B_s(\mathbf{p}) \cap \Phi) \subseteq \text{conv}(B_s(\mathbf{p}) \cap \Phi) \). Likewise, we can prove the same conclusions hold for \( \Phi \), which is obtained by aggregating elements in each generator’s \( B_s(\mathbf{p}) \).

We refer the readers to [11] for a general proof of the concavity of the dual. The concavity and piecewise linearity of \( \bar{L}(\rho) \) for the present problem is shown next. Note that it is likely that such a result may have been proved elsewhere, given its simplicity, yet we have no precise reference.

**Lemma 9.** Consider \( \bar{L}(\rho) \) as defined in Def. 4. Then \( \bar{L}(\rho) \) is a concave and piecewise linear function of \( \rho \) with a finite number of nondifferentiabilities.

**Proof:** By definition, \( \bar{L}(\rho) \) is given by

\[
\begin{align*}
\min_{(\mathbf{u}_s, \mathbf{p}_s) \in \mathcal{X}_s} & \left\{ \sum_{s=1}^{S} f_s(\mathbf{u}_s, \mathbf{p}_s) + \rho^T(d - \sum_{s=1}^{S} \mathbf{p}_s) \right\} \\
= & \rho^T \mathbf{d} + \sum_{s=1}^{S} \min_{(\mathbf{u}_s, \mathbf{p}_s) \in \mathcal{X}_s} \left\{ f_s(\mathbf{u}_s, \mathbf{p}_s) - \rho^T \mathbf{p}_s \right\} \\
= & \rho^T \mathbf{d} + \sum_{s=1}^{S} \min_{(\mathbf{u}_s, \mathbf{p}_s) \in \mathcal{X}_s} \left\{ f_s(\mathbf{u}_s, \mathbf{p}_s) - \rho^T \mathbf{p}_s \right\},
\end{align*}
\]

where the last equality holds since the optimum occurs at least one of the vertices. Note for any given element in \( \Phi_s \), \( f_s(\mathbf{u}_s, \mathbf{p}_s) - \rho^T \mathbf{p}_s \) becomes an affine function with respect to \( \rho \). Therefore \( \bar{L}(\rho) \) is a point-wise minimization of a finite number of affine functions, which is concave and piecewise.
linear. Furthermore, due to the finiteness of $\Phi_s$, $L(\rho)$ has a finite number of nondifferentiabilities.

The maximization of a concave and piecewise linear function may be formulated as an LP, whose optimum, if it exists, is always attained at extreme points; however, the constraint set requires enumerating all possible extreme points and such an avenue is generally inadvisable.

**Proposition 10** (Necessary and Sufficient Optimality Conditions for Convex Hull Price). Given a demand vector $d$, $\rho$ is a convex hull price for $d$ if and only if

$$d \in \text{conv}(B(\rho)).$$  

*Proof:* From Lemma 9, for any element in $\Phi_s$, $f_s(\hat{u}; \overline{p}) - \rho^T \overline{p}$ is an affine function of $\rho$ with gradient $-\overline{p}$. Also, because set $\Phi_s$ is finite, by Corollary 2.6 in [14], we obtain

$$\partial L(\rho) = d - \sum_{s=1}^{S} \text{conv}\{\overline{p}_s | \overline{p}_s \in B_s(\rho)\}$$

$$= \{d - \hat{d} \in \text{conv}(B(\rho))\}.$$  

The necessary and sufficient conditions for $\rho$ to be optimal are $0 \in \partial L(\rho)$ [15], which is equivalent to $d \in \text{conv}(B(\rho)).$

**IV. AN EXTREME-POINT SUBDIFFERENTIAL ALGORITHM**

Based on the analysis in Section III, it emerges that a convex hull price, denoted by $\rho$, is obtained when the load vector lies in the convex hull of $B(\rho)$. Intuitively, if this were not the case, then this suggests a method in which a sequence of convex hulls of the aggregate auction-surplus-maximization quantity set is constructed such that in the limit, the convex hull will contain the demand vector. Note that a new element of this sequence is based on updates of the price vector. In Subsection IV-A, we provide an outline of the EPSD algorithm and how search directions for $\rho$ may be efficiently computed. In Subsection IV-B, an overview of steplength choice and backtracking schemes is provided. The ESPG method is characterized by finite-termination behavior. We refer the readers to the Part II of this paper for illustrative examples and convergence analysis.

**A. Description of the EPSD scheme**

In iterative schemes for nonlinear programming, where one is often trading off between feasibility and optimality, one often uses a merit function to obtain a measure of progress [16]. In this particular instance, the iterates are always feasible and the merit function is effectively a measure of the departure from optimality. We consider a merit function given by the Euclidean distance between the demand and the convex hull of the aggregated auction-surplus-maximization quantity set. Notably, this merit function is nonnegative for any iterate and is zero at optimality.

**Definition 11** (Merit function). The distance between $d$ and $\text{conv}(B(\rho))$, denoted by $\gamma(\rho)$, is defined as

$$\gamma(\rho) \triangleq ||d^p(\rho) - \hat{d}||.$$  

where $d^p(\rho)$ is the projection of $d$ onto $\text{conv}(B(\rho))$, and defined as

$$d^p(\rho) \triangleq \arg \min_{d(\rho) \in \text{conv}(B(\rho))} ||d - \hat{d}||.$$  

Note the value of the merit function $\gamma(\rho)$ is uniquely determined by a quadratic program whose constraint set corresponds to the bundle of cuts that exact capture the value of the objective function at $\rho$. We propose to update the prices along the direction of vector $d - d^p(\rho)$, which is a descent direction associated with the merit function $\gamma(\rho)$. Surprisingly, this direction also represents the steepest ascent direction of $L(\rho)$, as the next proposition shows. More specifically, problem (T1) is closely related to problem (1.2.1) on Page 347 in [10] and Proposition 12 follows from Theorem 1.2.2 and Corollary 1.2.3 on Page 349 in [10].

**Proposition 12.** Suppose $d(\rho)$ is defined as (11). Then the vector $d - d^p(\rho)$ is the steepest ascent direction of $L(\rho)$.

*Proof:* See Theorem 1.11 in [15].

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Subgradient methods appear to have been first developed in the 70s by Shor [15] and can be combined with a host of steplength rules. An oft-used rule is a steplength sequence that increases.

In standard differentiable nonlinear programs, the gradient direction is not cheaply computable; instead, we propose to compute the steepest ascent direction by solving the projection problem in (11). This is generally a challenging task since it requires the computation of $\text{conv}(B(\rho))$. By computing $\text{conv}(B(\rho))$, the subdifferential of the piecewise-linear objective function is implicitly obtained through the extreme points. To facilitate this computation, we employ the extreme points of $\text{conv}(B(\rho))$. In order to further reduce the computational burden, we obtain the extreme points of $\text{conv}(B(\rho))$ as an aggregation of the extreme points of $\text{conv}(B(\rho))$.  

Define $\{p_{sk}\} = B_s(\rho) \cap \Phi_s$ with cardinality $K_s$, then by Lemma 8, it contains all extreme points of $\text{conv}(B(\rho))$. Through this decomposition, we solve $S$ smaller MIP subproblems, one for each generator, in each iteration instead of a large MIP problem over $S$ generators. Such decomposition is analogous to that utilized during a Lagrangian relaxation based on the separability across generators. Note that an extreme-point representation of $\text{conv}(B(\rho))$ may incur significant computational effort since the number of extreme points may grow to an exponential level as the the number of generators increases. We further leverage on the fact that the aggregation of convex hulls preserves separability across generators: The resulting projection problem can be cast as the following convex quadratic program.

$$\min_{\lambda_s, \forall s, k} ||d - \sum_{s=1}^{S} \sum_{k=1}^{K_s} \lambda_{sk} p_{sk}||^2$$

$$st: \sum_{k=1}^{K_s} \lambda_{sk} = 1, \forall s$$

$$\lambda_{sk} \geq 0, \forall s, k.$$  

The solution to the above problem provides us with the requisite projection. Essentially, we do not need to construct $B(\rho)$ explicitly, since we are only interested in its convex hull.
As shown later, this avenue avoids the need to contend with a direct specification of $B(\rho)$, a combinatorially challenging task.

**Proposition 13.** Suppose $\{\lambda_{sk}\}$ solves (12) for $k = 1, \ldots, K^s$ and $s = 1, \ldots, S$. Then $d^P(\rho^*)$ a solution of (11), is given by

$$d^P(\rho^*) = \sum_{s=1}^{S} K_s \sum_{k=1}^{K_s} \lambda_{sk} p_{sk}.$$  

**Proof:** By the commutability of the convex hull and sum operation, we have

$$\text{conv}(B(\rho)) = \text{conv} \left( \left\{ \sum_{s=1}^{S} \rho_s p_s \mid p_s \in B_s(\rho), \forall s \right\} \right)$$

$$= \left\{ \sum_{s=1}^{S} p_s \mid p_s \in B_s(\rho), \forall s \right\}$$

$$= \left\{ \sum_{s=1}^{S} K_s \sum_{k=1}^{K_s} \lambda_{sk} p_{sk} \mid \forall s; \lambda_{sk} \geq 0, \forall s, k \right\},$$

(13)

where the third equality holds by Lemma 8 and the fourth equality holds by the definition of $\{p_{sk}\}$. \qed

Note if $\{\lambda_{sk}\}$ solves (12), then for a given $s$ and $k$, $\lambda_{sk} > 0$ or $\lambda_{sk} = 0$. We denote these strictly positive ones by $\{\lambda_{sk}\}$ and their associated coefficients in (12) by $\{\tilde{p}_{sk}\}$, and define

$$\{\tilde{d}\} = \sum_{s=1}^{S} \tilde{p}_{sk}.$$  

(14)

It is obvious that $d^P(\rho^*) \in \text{conv}(\{\tilde{d}\})$, because we essentially just eliminate the zero terms in the optimal objective function of (12).

The number of decision variables in (12), $\sum_{s=1}^{S} K_s$, increases at a linear rate with respect to the number of generators. Given the offers from electricity markets under normal conditions, $K_s = 1$ for most generators: due to the large distinctions of different generating technologies in terms of economic merits and operational flexibility, most generators are either operated at the maximum, or priced themselves out of the markets. Accordingly, the scale of (12) remains small and can be easily solved. We are now ready to state our algorithm. Note that the steplength selection scheme (Algo. 15) is described in the next subsection.

**Algorithm 14** (Extreme-point Subdifferential Scheme (EPSD)).

1: Initialization: price $\rho^1$, max steplength $c > 0$; iteration index $\nu = 1$; merit measure $\gamma^0 = \infty$; search direction $\Delta^0 = \mathbf{0}$; Tolerance: $\epsilon$.
2: while $\gamma^\nu - 1 \geq \epsilon$ do
3: for $s = 1 : S$ do
4: Obtain $B_s(\rho^\nu) \cap \Phi_s$ from (6);
5: end for
6: Compute $d^P(\rho^\nu)$ from (12);
7: Compute $d^\nu / \gamma^\nu$ from (14);  
8: Set $\gamma^\nu = \|d - d^P(\rho^\nu)\|_2$;
9: if $\gamma^\nu < \gamma^\nu - 1$ or $d^P(\rho^\nu) = d^P(\rho^\nu - 1)$ then
10: Phase I:
11: $\alpha^\nu = c$, $\Delta^\nu = \frac{d - d^P(\rho^\nu)}{\|d - d^P(\rho^\nu)\|_2}$;
12: $\rho^{\nu+1} = \rho^\nu + \alpha^\nu \Delta^\nu$, $\nu = \nu + 1$;
13: else
14: Phase II:
15: $[\rho^\nu, \gamma^\nu, d^\nu, \nu]$  
16: = Backtrack($\Delta^\nu = 1$, $\rho^{\nu - 1}$, $\Delta^\nu - 1$, $\nu^\nu - 1$, $\gamma^\nu$, $\nu^\nu - 1$);  
17: end if
18: end while

It should be recalled that the merit function $\gamma$ is determined by the bundle of cuts that captures the exact value of the point in question. In a non-degenerate setting, a decrease in $\gamma$ implies a jump from one face of the dual function to a strictly better face. In this instance, the elements of the previous underlying bundle are totally discarded. On the other hand, an unchanged value of $\gamma$ implies that the underlying bundle has remained unchanged, and suggests that the two points in question are at either the same face or the same edge of the polytope. Consequently, it is reasonable to keep searching over the same (steepest) ascent direction.

**B. Steplength Selection**

Given a projection direction $\Delta^\nu$, the key question is how one may choose the steplength. Standard subgradient schemes rely on diminishing steplength rules that require a steplength requirement is not guaranteed when an arbitrary subgradient is selected. Given such a property, the EPSD method conducts a linesearch along the obtained search direction to ensure a monotonically decreasing $\gamma$. This search process could be a simple one, based on reducing the steplength by specific amounts. Alternately, it could be a more intricate process that relies on using function values to develop a model of the function and subsequently minimizing this model over the steplength [16].

In this setting, a user-specified upper bound, denoted by $c$, is chosen as the point from which the steplength may be reduced until sufficient descent is made with respect to a suitably defined merit function.

**Algorithm 15** (Backtrack($\Delta^\nu = 1$, $\rho^{\nu - 1}$, $\Delta^\nu - 1$, $\nu^\nu - 1$, $\gamma^\nu$, $\nu^\nu - 1$)).

1: Initialization: $\text{flag} := \nu - 1$;
2: while $\gamma^\nu \geq \gamma^\nu - 1$ do
3: Obtain $\alpha$ by solving
4: $$(\rho^\nu + \alpha(\Delta^\nu))^T \tilde{d} - v(\tilde{d})$$
5: $$= (\rho^\nu + \alpha(\Delta^\nu))^T \tilde{d} - v(\tilde{d})$$
6: $$\rho^{\nu+1} := \rho^\nu + \alpha \Delta^\nu.$$
generation capacity could be used to provide either energy and power. The merit function has increased. In fact, this implies that the new iterate has overshot the current face and entered a new face. In this case, the faces of $L(\rho)$ crossed by this overshooting are investigated in the hope of reducing $\gamma$. From the perspective of the underlying bundle of cuts, the backtracking procedure looks for points whose bundle contains both the set of exact cuts of the flag-th iterate and that of the $\nu$-th iterate, excluding those cuts corresponding to the inactive constraints in the quadratic program to computing $\gamma$. Specifically, the search process entails determining the steplength required to reach the edges common to neighboring faces. This is effectively a backtracking process but one in which the steplength is not reduced by a constant ratio; in fact, the steplength takes values in accordance with the distance of the edges from the current iterate. When descent in the merit function is achieved, the backtracking subroutine terminates. Finally, it is worth noting that $\nu$, the index, is incremented during the backtracking phase as well; in effect, upon termination, $\nu$ specifies the total number of steps, both standard and backtracking, that the algorithm has produced. Algorithm 15 makes the steps of the subroutine precise and the next example illustrates these ideas.

V. ENERGY-RESERVE CO-OPTIMIZED MARKETS

Power systems are prone to failures of generators and transmission lines, as well as unanticipated changes in demand. In order to maintain system reliability, capacity which can be quickly dispatched is procured. The capacity is referred to as “reserves”, and may be viewed as an insurance policy against contingencies. Typically, reserve requirements are imposed on the system operation and these requirements introduce market mechanisms for reserves. Early generations of electricity markets traded reserves separately from energy markets while more recently several markets trade both energy and reserves. In these markets, energy, and reserves are cleared simultaneously to minimize the total as-bid costs. In this paper, we focus on the ramp-up spinning reserves, which can be provided by online generators. We bound the available reserves that generator $s$ may provide at period $h$ as

$$r_s \leq u_{s,h} \min(p_s^{\text{max}}, p_s^{\text{max}} - p_{s,h}),$$  

(15)

where $p_s^{\text{max}}$ is a physical characteristic of the generator based on its ramping capacity, $p_s^{\text{max}}$ denotes the generator’s capacity and $p_{s,h}$ denotes the energy generating level. This indicates that generation capacity could be used to provide either energy or reserve, but not both. A salient feature of reserves is that it shares the capacity constraint with energy.

Whether to impose the reserve requirement as an equality constraint or inequality constraint is a market design issue. When the requirement is modeled as equality constraint, the extension from the energy only market to the integrated market is relatively straightforward, as the underlying mathematical programming model remains unchanged. However, if inequality constraints of reserve requirement are imposed, this leads to a non-negativity bound on the resulting reserve prices in the Lagrangian dual problem. However, we proceed to show that the EPSPD method always produces nonnegative reserve prices without the direct introduction of nonnegativity bounds.

Depending on the market design, the generator may or may not be allowed to submit price and availability offers for reserve [17]. One alternative lies in allowing generators to submit price and availability offers for reserves $f_e^*(u_s, x_s) \geq 0$, while the available reserve are not necessarily be cleared.

Definition 16 (Co-optimization Model with explicit reserve offers). The Co-optimization Model with explicit reserve offers is defined as

$$\begin{align*}
\min_{\rho, P_s, \forall s} \sum_{s=1}^{S} \left( f_s^e(u_s, p_s) + f_s^r(x_s, e_s) \right) \\
\text{s.t.} \quad \sum_{s=1}^{S} P_s = d^e, \\
\sum_{s=1}^{S} x_s \geq d^r, \\
(u_s, p_s) \in X_s, \quad \forall s, \\
r_{s,h} \geq 0, \quad \forall s, h, \\
r_{s,h} \leq u_{s,h} \min(p_s^{\text{max}}, p_s^{\text{max}} - p_{s,h}), \forall s, h.
\end{align*}$$

(16)

The other option is to impose an obligation on all generators to provide all available reserves, the underlying rationale being that reserves involves little, if any, variable cost but much higher opportunity costs.

Definition 17 (Co-Optimization Model without reserve offers). The Co-Optimization Model without reserve offers is defined as

$$\begin{align*}
\min_{\rho, P_s, \forall s} \sum_{s=1}^{S} f_s^e(u_s, p_s) \\
\text{s.t.} \quad \sum_{s=1}^{S} P_s = d^e, \\
\sum_{s=1}^{S} x_s \geq d^r, \\
(u_s, p_s) \in X_s, \quad \forall s, \\
r_{s,h} = u_{s,h} \min(p_s^{\text{max}}, p_s^{\text{max}} - p_{s,h}), \forall s, h.
\end{align*}$$

(17)

We begin our analysis by considering the following problem:

Definition 18 (Computational Model). The Computational Model is defined as

$$\begin{align*}
\min_{\rho, P_s, \forall s} \sum_{s=1}^{S} \left( f_s^e(u_s, p_s) + f_s^r(x_s, e_s) \right) \\
\text{s.t.} \quad \sum_{s=1}^{S} P_s = d^e, \\
\sum_{s=1}^{S} x_s = d^r, \\
(u_s, p_s) \in X_s, \quad \forall s, \\
r_{s,h} \geq 0 \quad \forall s, h, \\
r_{s,h} \leq u_{s,h} \min(p_s^{\text{max}}, p_s^{\text{max}} - p_{s,h}), \forall s, h.
\end{align*}$$

(18)
Note the only difference between the (18) and (16) is that the inequality constraints on reserve requirement are replaced by equalities.

Proposition 19. Given nonnegative reserve offers \( f_s(U_s, L_s) \) and nonnegative reserve requirements \( d_s \), models (18) and (16) yield the same convex hull prices.

Proof: Note that both models share the same dual problem, with the exception that model (16) is subject to nonnegativity bounds on the dual variables. It suffices to show that the reserve prices from model (18) are always nonnegative.

Also note for any given energy allocation, the reserve sold by an on-line generator in both models can vary from 0 to \( \min(r_s^{\text{max}}, p_s^{\text{max}} - p_s,h) \) without violating any individual constraint. Accordingly, if the reserve price at some hour is less than zero, then all generators will supply zero reserves to maximize their auction surplus.

Suppose, at the optimum, the reserve price at some hour is less than zero from model (18), then the reserve requirement \( d_s \) must belong to the convex hull of the best response set. However, \( d_s > 0 \) while the best response of all generators is to provide zero reserve and we have a contradiction. Therefore, reserve prices from model (18) are always nonnegative.

Proposition 20. Consider models (18) and (17). Then, the convex hull price arising from (18) with zero reserve offer price is identical to the convex hull price arising from (17).

Proof: Suppose \( f_s(U_s, L_s) = 0 \) in model (16) and (18). From Proposition 19, we know that models (16) and (18) shares the same convex hull prices, therefore, a sufficient condition for this result is that models (16) and (17) have a common dual function. The remainder of the proof shows that this is indeed the case.

Suppose the reserve price is strictly positive. Then for any given energy allocation, an on-line generator in models (16) and (17), can choose any reserve from \( [0, \min(r_s^{\text{max}}, p_s^{\text{max}} - p_s,h)] \) and \( \min(r_s^{\text{max}}, p_s^{\text{max}} - p_s,h) \), respectively. However, \( \min(r_s^{\text{max}}, p_s^{\text{max}} - p_s,h) \) maximizes auction surplus and such a choice is feasible for both models. Furthermore, since the dual functions in both instances are continuous, they assume the same value at zero reserve prices.

As shown in Proposition 19 and Proposition 20, by allowing availability offers, the solution of the constrained dual problem coincides with that from an unconstrained problem for which our EPSD method can apply. Note such conclusions only apply to pricing rather than the allocation: the solutions to (16), (17), and (18) may differ, despite yielding the same convex-hull prices.

VI. ECONOMIC ANALYSIS

In this section, we analyze outcomes of energy only and energy-reserve co-optimized markets under different pricing and uplift mechanisms. Two uplift mechanisms are considered. Make-whole uplift pays the difference between an individual generator’s price-based revenues and its as-bid costs. Opportunity-cost uplift compensates the difference between the realized surplus of an individual generator following the ISO’s schedule and the maximal auction surplus if it were operated otherwise, assuming the same prices. Although the convex-hull pricing and opportunity-cost uplift are usually discussed collectively, we do explore the possibility of combining either pricing mechanism (convex-hull or marginal-cost) with either uplift scheme (opportunity-cost or make-whole).

Case 1 uses the 26-unit data from [18], while case 2, 3, 4 and 5 are generated based on case 1 with (1) perturbed offer prices, and (2) duplicated units and scaled demand, as one, two, three and four times of the original system, respectively. By doing this, we construct test systems with up to 104 units.

We compare the total payments and the uplift payments for the test systems under different pricing and uplift schemes, assuming the same set of offers. We use bars to denote the total payments and lines for uplift payments under different payment mechanisms in Figure 2. As the plot shows, uplift payments are at least two orders of magnitude less than the total payments. Thus, the differences in total payments under different schemes are mainly due to the change of pricing rules. The maximal deviation in total payment for the same system is up to 5% in our tests, and it is unclear if marginal-cost pricing or convex-hull pricing lead to higher payments in general. While uplift payments may represent a
relatively small percentage of total energy costs, as shown in the test cases, the difference in revenues between the diverse mechanisms may be non-trivial. Further, the impact of changing mechanisms may have contrasting effects across stakeholders. Indeed, some generators, specifically marginal and block-loaded, rely heavily on uplift payments. In a competitive regime, inherently selfish firms are motivated by profits, implying that the underlying pricing and allocation mechanisms are of prime concern. Given the non-convexities and discreteness in the electricity markets, there appears to be no “correct” sensitivity metric available (which in turn corresponds to prices) and both mechanisms are essentially based on approximations.

The total payments increase in proportion to the size of the system: the ratios of total payments between the 104-unit system and the 26-unit system vary from 3.92 to 4.05. On the other hand, the uplift payments, except the opportunity-cost uplift under marginal-cost pricing, grow at a much slower pace. It is worth noting that opportunity-cost uplift payments under convex-hull pricing tend to decrease as systems are scaled up. By definition, opportunity-cost uplift payments are always no less than the make-whole uplift payments. The plots reaffirm this fact and indicate the opportunity-cost uplift payments may be much higher than the make-whole uplift payments, especially for co-optimized markets with marginal-cost pricing. Indeed, in co-optimized markets, more capacity is called on-line to provide reserves, leading to higher marginal-cost prices. The high prices result in less or even zero make-whole uplift payment. However, under such high prices, the commitment and dispatch schedules are not optimal from the individual player’s view point, which requires the opportunity-cost uplift payments. Also note the convex-hull pricing minimizes the opportunity-cost uplift, rather than the make-whole uplift payment.

VII. Conclusions

Convex hull pricing has been introduced recently for electricity markets. While this technique results in a concave optimization problem to determine prices, the objective function is not differentiable. In this paper, we have introduced an extreme-point subdifferential algorithm to efficiently compute prices. The finite-termination property and numerical performance of the EPSPD method is presented in Part II of this paper. A prescription of prices is a key component of any market design. Extant designs require ex-post uplift payments that may be difficult to justify, and the proposed convex hull pricing appears to be far more elegant. However, further research is needed to understand the possible outcomes of the market. Along these directions, many avenues open for future research:

(i) Transmission constraints. How do constraints impact prices when incorporated in the current congestion management and transmission rights mechanisms?
(ii) Market power. The opportunity-cost uplift payments are provided to both on-line and off-line generation units. Does this imply greater opportunities to exercise market power?

(iii) Incentives. Is it not clear whether the new pricing mechanism will send better signals to attract investment, and encourage generation expansion. Is this pricing scheme sufficient for generators? What are the actual risks and benefits in a realistic setting?

All of these topics will be considered in current research.

REFERENCES


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